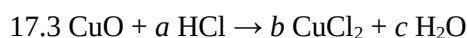


Row operations

1. We have defined three row operations. Suppose you have an augmented matrix and the corresponding system of linear equations. Describe how row operations equate to corresponding operations on the equations.

2. Suppose you have the following reaction where you are reacting 17.3 moles of copper oxide with hydrogen chloride.



and the equations from H, Cu, O and Cl, in that order, result in the following system of four equations in three unknowns (the unknowns being a , b and c , in that order):

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & -1 & 0 & -17.3 \\ 0 & 0 & -1 & -17.3 \\ 1 & -2 & 0 & 0 \end{array} \right)$$

Add -1 times Row 1 onto Row 4, then add -2 times Row 2 onto Row 4, and then add 2 times Row 3 onto Row 4. Is there a unique solution, and if so, what is it?

If you were to first swap Rows 1 and 4, then add -1 times Row 1 onto Row 4, then add 2 times Row 2 onto Row 4, and then add -2 times Row 3 onto Row 4. Is there a unique solution, and if so, do you expect the solution to be the same as the previous solution?

3. If you performed the row operations in yet a different order, you could end up with the following matrix that is in row-echelon form. The solution to this is identical to the solutions you found in Question 2:

$$\left(\begin{array}{ccc|c} 1 & 0 & -2 & 0 \\ 0 & -2 & 2 & 0 \\ 0 & 0 & -1 & -17.3 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

Your peers who are taking linear algebra from the faculty of mathematics have a different definition of row-echelon form, whereby they require the leading non-zero entry to be “1”, so any row that has a leading non-zero entry must be divided by that leading non-zero entry. Consequently, multiply Row 2 by -0.5 and multiply Row 3 by -1 . Does this result in the same solution?

The reason we do not use the faculty of mathematics definition of row-echelon form is that if you require the leading non-zero entry to be a “1”, this will result in approximation $n^2/2$ addition floating-point divisions if the rank of the matrix is n . If you don’t divide each row by the leading non-zero entry, then when you do backward substitution, then you will only have to do one additional division per row for a total of n divisions.

Suppose we are finding a unique solution to a system of 1000 equations in 1000 unknowns, and the rank is $n = 1000$. Compare $1000^2/2$ and 1000, and which is likely the more intelligent choice?